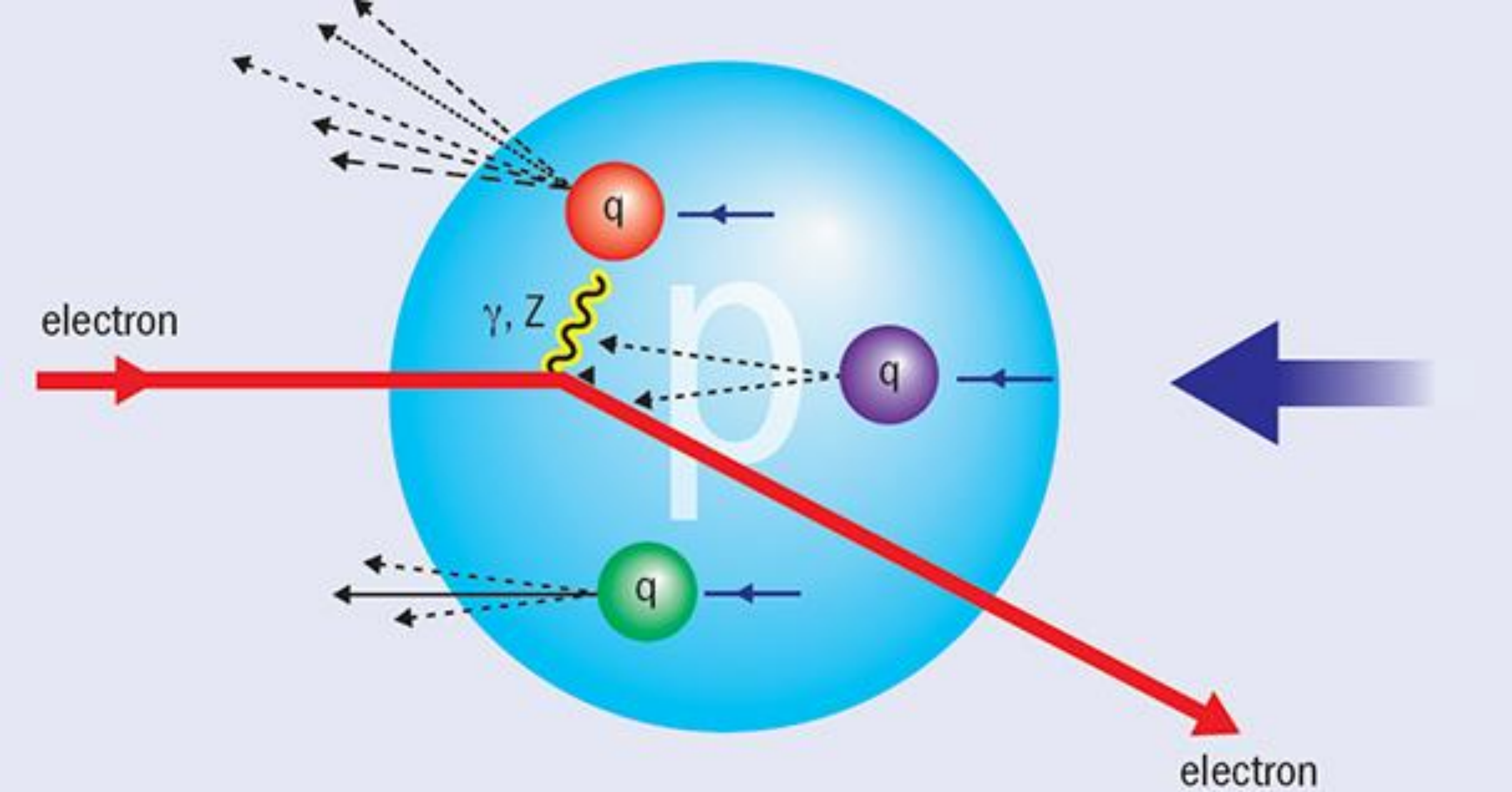

Particle Physics I

Lecture 10: Deep inelastic electron-proton scattering

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Short recap and learning targets

- **Ultimate goal:** investigate high-energy electron-proton inelastic scattering where the proton breaks up in the interaction, referred to as Deep Inelastic Scattering (DIS)

Learning targets

- General Lorentz-invariant (LI) extension of the $e^- p \to e^- p$ elastic scattering with Form Factors (FFs) replaced by structure functions
- Describe DIS in terms of QED interaction of a virtual photon with the constituent quarks inside the proton
- Interpret the experimental data in terms of the quark-parton model
- Relation between the structure functions and the parton distribution function that describe the momentum distribution of the quarks

e^-p elastic scattering at very high q^2

- The **Rosenbluth formula** for e^-p elastic scattering:
  - $G_E(q^2)$: related to the charge distribution inside the proton
  - $G_M(q^2)$: related to the distribution of the magnetic moment of the proton

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{elastic}} = \frac{\alpha^2}{4E_1^2\sin^4\theta/2}\cdot\frac{E_3}{E_1}\left(\frac{G_E^2+\tau G_M^2}{1+\tau}\cos^2\frac{\theta}{2}+2\tau G_M^2\sin^2\frac{\theta}{2}\right)$$

with the Lorentz-invariant quantity $\tau = -q^2/4M^2$, at very high q^2 we have $\tau \gg 1$ which leads to

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{elastic}} = \frac{\alpha^2}{4E_1^2\sin^2\theta/2}\cdot\frac{E_3}{E_1}\cdot 2\tau G_M^2$$

$e^- p$ elastic scattering at very high q^2

- The **Rosenbluth formula** for $e^- p$ elastic scattering at high q^2:

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{elastic}} = \frac{\alpha^2}{4E_1^2 \sin^2\theta/2} \cdot \frac{E_3}{E_1} \cdot 2\tau G_M^2$$

- From $e^- p$ elastic scattering experiments we found that the proton magnetic Form Factor is

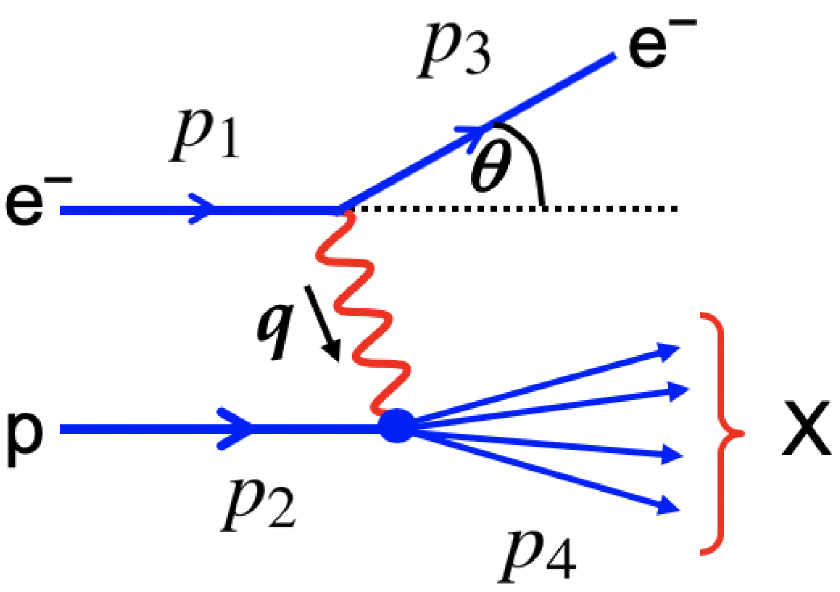
$$G_M^p(q^2) \approx \frac{1}{\left(1 + \frac{q^2}{0.71\text{GeV}^2}\right)^2} \propto q^{-4} \text{ at high } q^2$$

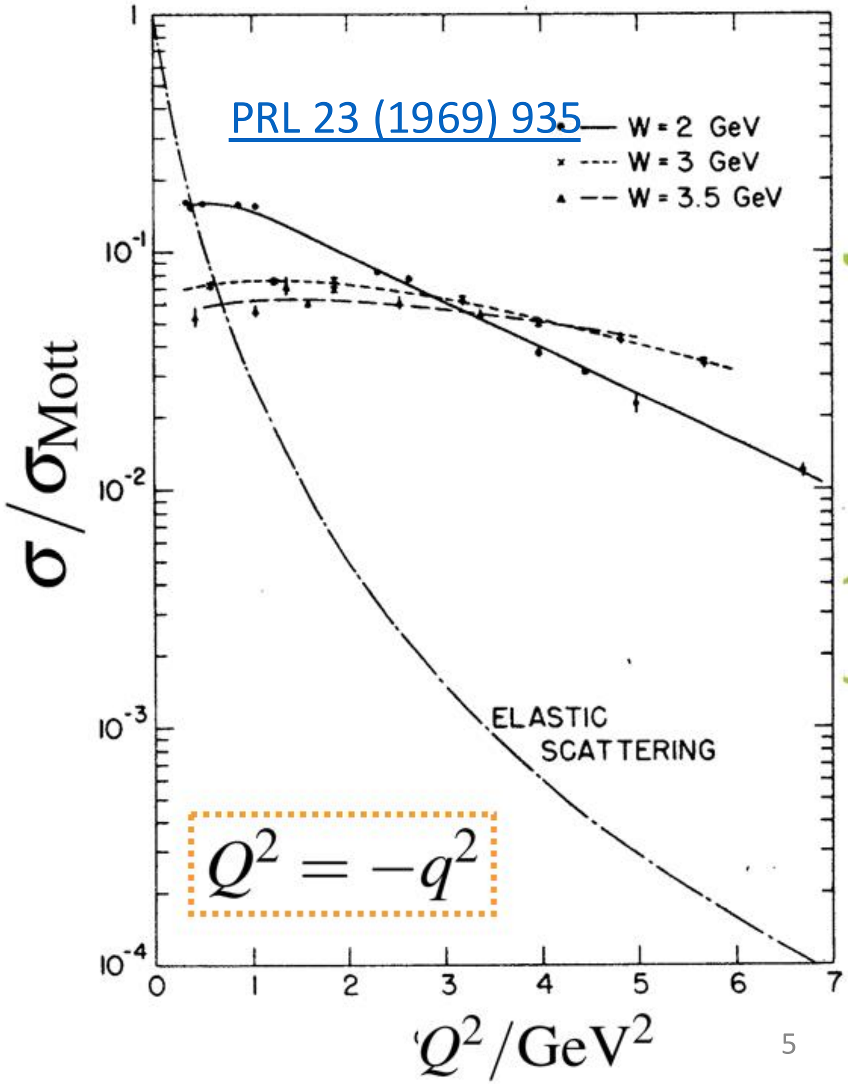
$$\Rightarrow \left(\frac{d\sigma}{d\Omega}\right)_{\text{elastic}} \propto q^{-6}$$

$e^- p$ elastic scattering at very high q^2

- Due to the finite proton size, **at high q^2** elastic scattering is unlikely and inelastic interactions where the proton breaks up dominate

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{elastic}} \propto q^{-6}$$





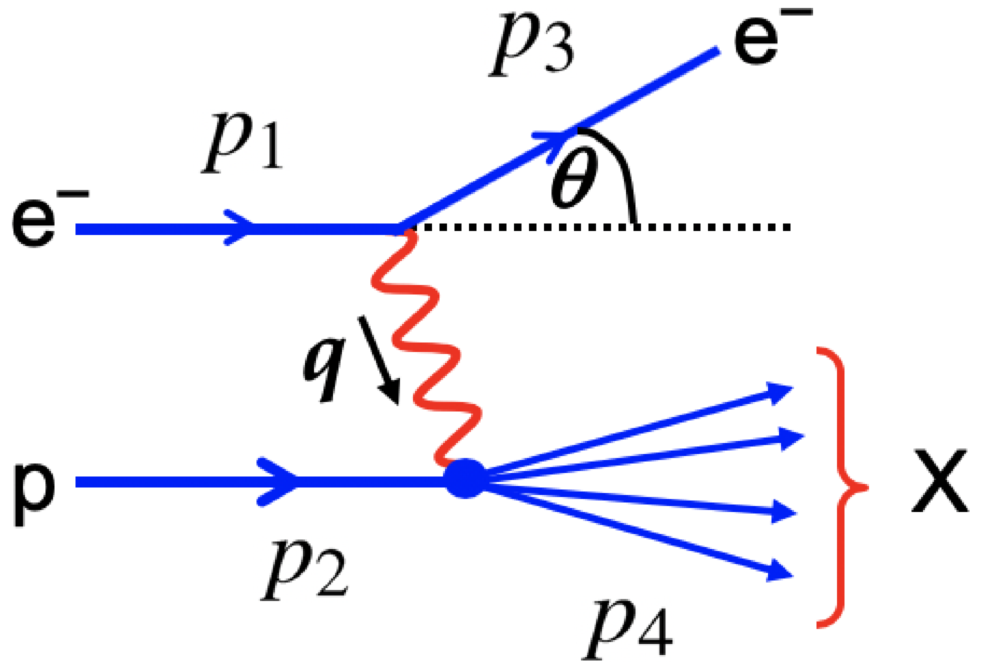
$$M_X^2 = W^2 = p_4^2 = (p_1 + p_2 - p_3)^2 = (q + p_2)^2$$

Kinematics of $e^- p$ inelastic scattering

- For inelastic scattering the mass of the final hadronic system is no longer the proton mass m_p
- The final state must contain at least one baryon, implying that $M_X > m_p \Longrightarrow$ additional degree of freedom
- We introduce four useful **Lorentz-invariant** quantities: $\boldsymbol{x, y, \nu, Q^2}$

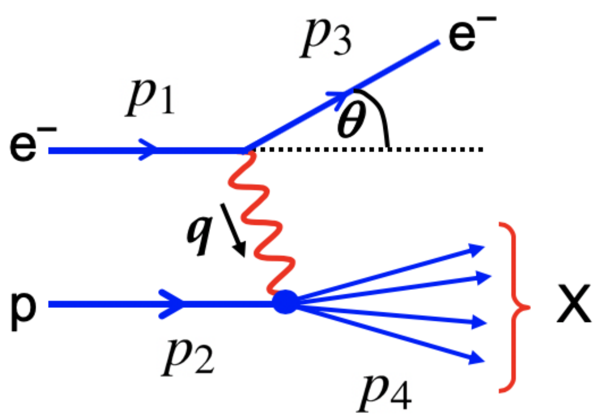
$$Q^2 \equiv -q^2, \qquad x \equiv \frac{Q^2}{2p_2 \cdot q}$$

$$y \equiv \frac{p_2 \cdot q}{p_2 \cdot p_1}, \qquad \nu \equiv \frac{p_2 \cdot q}{m_p}$$



$$M_X^2 = p_4^2 = E_4^2 - |\vec{p}_4|^2$$

Kinematics of $e^- p$ inelastic scattering: ν and Q^2



- Definition of the exchanged momentum Q^2:

$$Q^2 \equiv -q^2 = (p_1 - p_3)^2$$

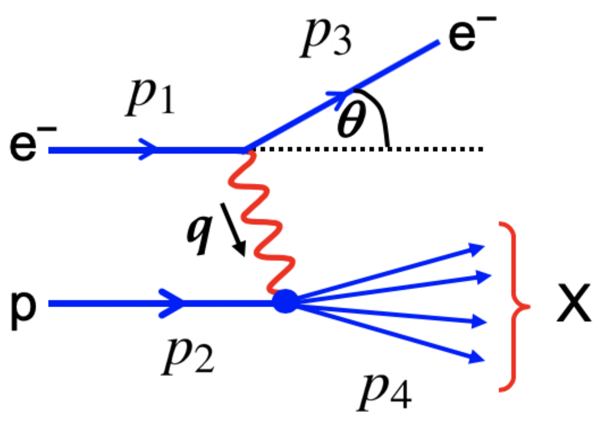
- $Q^2 = 4E_1 E_3 \sin^2 \theta/2$ (neglecting the electron masses)

- Definition of ν:

$$\nu \equiv \frac{p_2 \cdot q}{m_p}$$

- In the frame where the proton is at rest ν is simply the energy lost by the electron: $\nu = E_1 - E_3$

Kinematics of $e^- p$ inelastic scattering: Bjorken x



- Definition of Bjorken x:

$$x \equiv \frac{Q^2}{2p_2 \cdot q}, \text{where } Q^2 = -q^2 > 0$$

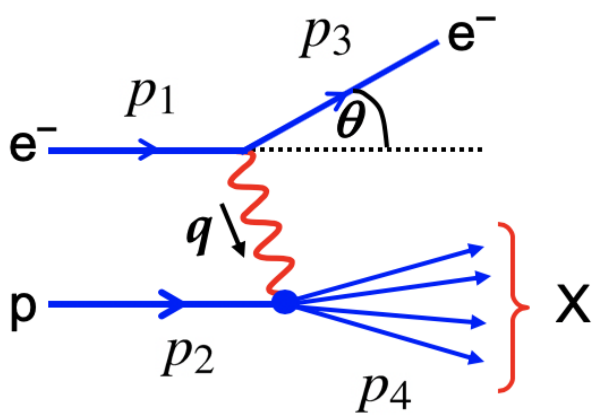
- We can derive the following expression for the mass of the hadronic system M_X (often W is used)

$$M_X^2 = p_4^2 = (q + p_2)^2 = -Q^2 + 2q \cdot p_2 + m_p^2$$

$$\Longrightarrow Q^2 = 2q \cdot p_2 + m_p^2 - M_X^2 \Longrightarrow Q^2 < 2q \cdot p_2 (M_X > m_p)$$

- We get $0 < x < 1$ for an inelastic process and $x = 1$ for an elastic one (the proton is intact, $M_X = m_p$)

Kinematics of e^-p inelastic scattering: inelasticity y



- Definition of the inelasticity y:

$$y \equiv \frac{p_2 \cdot q}{p_2 \cdot p_1}$$

- In the lab frame: $p_1 = (E_1, 0,0, E_1)$, $p_2 = (m_p, 0,0,0)$, $q = (E_1 - E_3, \vec{p}_1 - \vec{p}_3)$

$$y = \frac{m_p(E_1 - E_3)}{m_p E_1} = 1 - \frac{E_3}{E_1}$$

- y is the fractional energy loss of the incoming particle $0 < y < 1$

- In the CoM frame (after neglecting the electron and proton masses):

$p_1 = (E, 0,0, E)$, $p_2 = (E, 0,0, -E)$, $q = (E, E\sin\theta^*, 0, E\cos\theta^*)$

$$y = \frac{1}{2}(1 - \cos\theta^*) \text{ for } E \gg m_p$$

Relationship between kinematic variables

- We can rewrite the new kinematic variables in terms of square CoM energy for $e^- p$ collision s:

$$s = (p_1 + p_2)^2 = p_1^2 + p_2^2 + 2p_1 \cdot p_2 = 2p_1 \cdot p_2 + m_p^2 + m_e^2 (m_e \approx 0)$$

$$2p_1 \cdot p_2 = s - m_p^2$$

- For a fixed CoM energy, the kinematics of inelastic scattering can be described by any pair of the LI quantities $Q^2, \nu, x,$ and y (except ν and y)
  - for elastic scattering ($x = 1$) only one independent kinematic variable, the electron scattering angle θ

$$Q^2 \equiv -q^2, \qquad x \equiv \frac{Q^2}{2p_2 \cdot q}, \qquad y \equiv \frac{p_2 \cdot q}{p_2 \cdot p_1}, \qquad \nu \equiv \frac{p_2 \cdot q}{m_p}$$

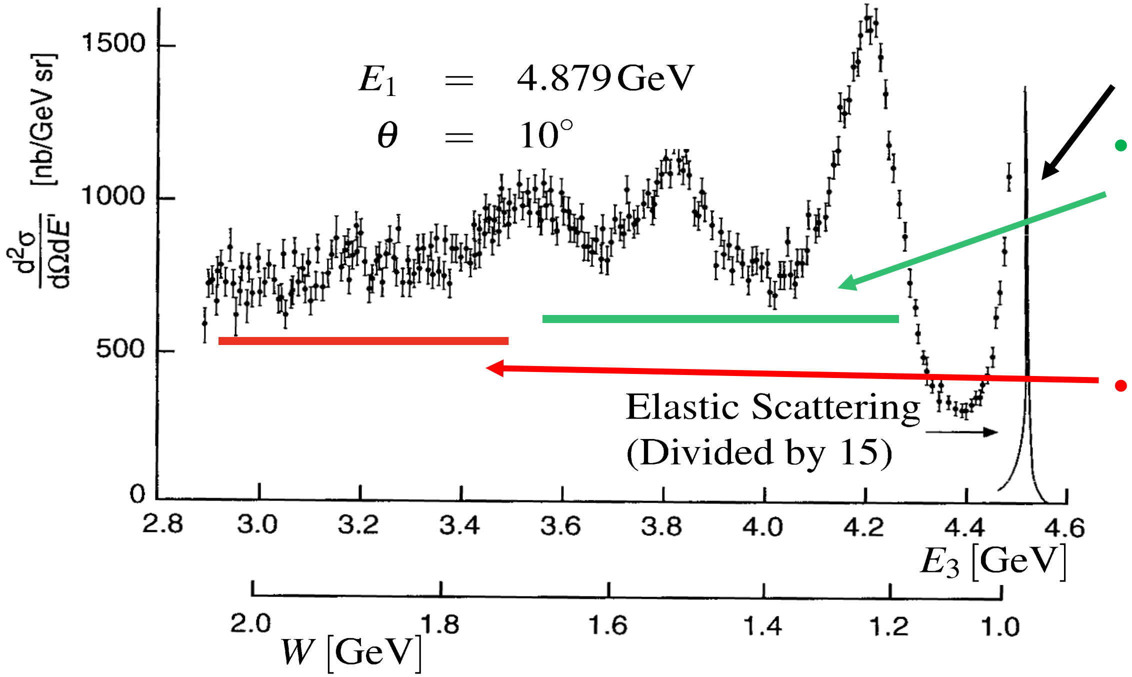
$$\Downarrow$$

$$x = \frac{Q^2}{2m_p \nu}, \qquad y = \frac{2m_p}{s - m_p^2}\nu, \qquad xy = \frac{Q^2}{s - m_p^2} \Longrightarrow Q^2 = \left(s - m_p^2\right) xy$$

Inelastic scattering: example

- Example: scattering at 4.879 GeV electrons from protons at rest
- Place a detector at 10° with respect to the beam axis and measure the energy of the scattered e^-
- Kinematics is completely determined from the electron energy and angle
- For this energy and angle the invariant mass of the final-state hadronic system is:

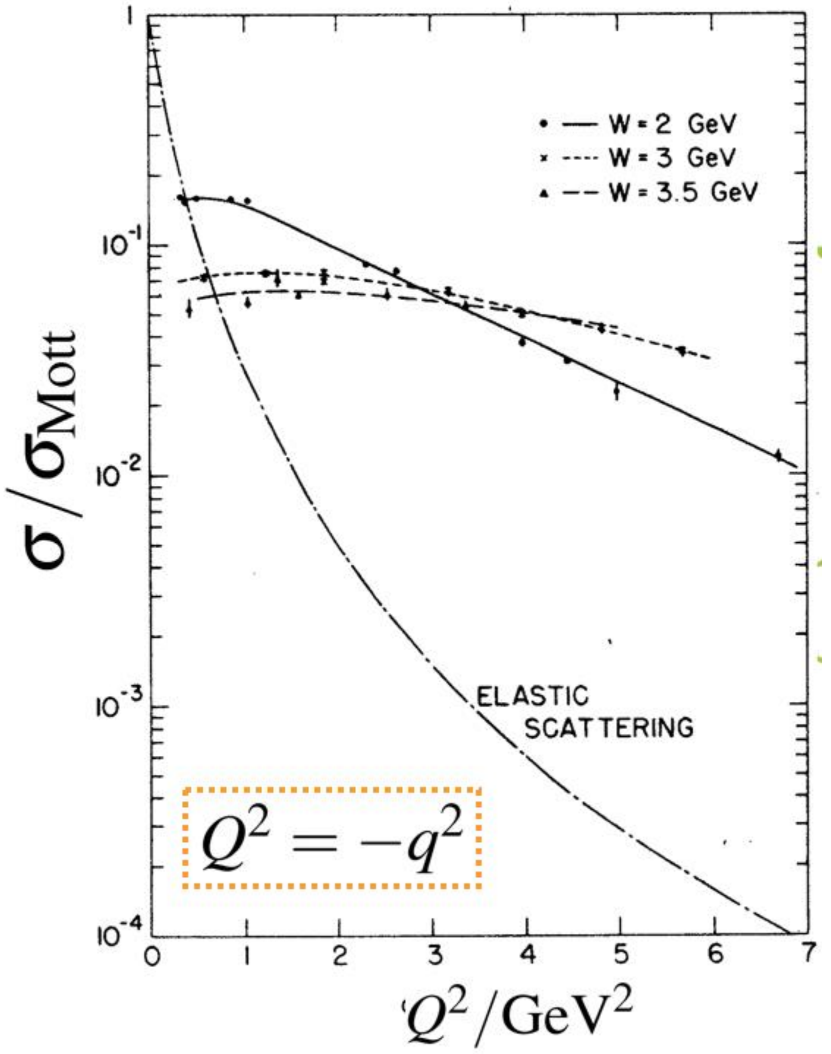
$$W^2 = M_X^2 = 10.06 - 2.03E_3$$



- **Elastic scattering**
  - proton remains intact
- **Inelastic scattering**
  - produce excited states of the proton (e.g. $\Delta^+(1232)$)
  - $W = M_\Delta$
- Deep Inelastic Scattering
  - proton breaks up resulting in a many-particle final state
  - **DIS = large *W***

Inelastic cross sections

- Repeat the experiment at different angles/beam energies and determine the q^2 dependence of the elastic and inelastic cross sections



- Elastic scattering falls off rapidly with q^2 due to the proton not point-like (e.g. Form Factors)
- Inelastic scattering cross section depends only weakly on q^2
- Deep Inelastic Scattering (DIS) cross section almost independent of q^2 ! (i.e. Form Factor → 1)

⟹ scattering from a point-like objects within the proton

Elastic → inelastic scattering

Elastic scattering:

- Only one independent variable (scattering angle θ)

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4\theta/2} \cdot \frac{E_3}{E_1}\left(\frac{G_E^2 + \tau G_M^2}{1+\tau}\cos^2\frac{\theta}{2} + 2\tau G_M^2 \sin^2\frac{\theta}{2}\right), \qquad \tau = Q^2/4m_p^2$$

Note: the energy of the scattered electron is determined by the angle θ!

- Using the LI kinematic variables, we can express the differential cross section in terms of Q^2

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4} \cdot \left[\frac{G_E^2 + \tau G_M^2}{1+\tau}\left(1 - y - \frac{m_p^2 y^2}{Q^2}\right) + \frac{1}{2}y^2 G_M^2\right]$$

which can be written as

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4} \cdot \left[f_2(Q^2)\left(1 - y - \frac{m_p^2 y^2}{Q^2}\right) + \frac{1}{2}y^2 f_1(Q^2)\right]$$

Elastic → inelastic scattering

Inelastic scattering:

- For DIS we have two independent variables ⟹ double differential cross section
- It can be shown that the most general Lorentz-invariant expression for $e^-p \to e^-X$ inelastic scattering (via an exchange of a single photon) is

$$\frac{d^2\sigma}{dx\mathrm{d}Q^2} = \frac{4\pi\alpha^2}{Q^4} \cdot \left[\left(1 - y - \frac{m_p^2 y^2}{Q^2}\right) \frac{F_2(Q^2,x)}{x} + y^2 F_1(Q^2,x) \right]$$

Elastic scattering:

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4} \cdot \left[\left(1 - y - \frac{m_p^2 y^2}{Q^2}\right) f_2(Q^2) + \frac{1}{2} y^2 f_1(Q^2) \right]$$

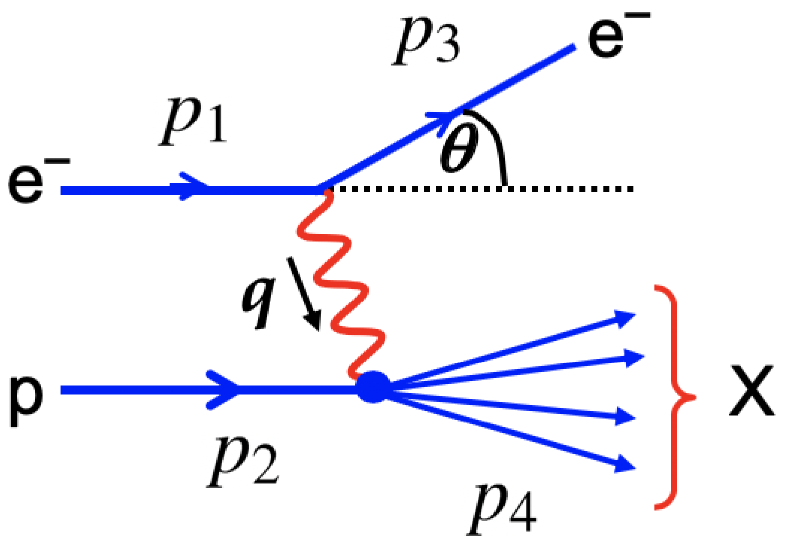
Deep Inelastic Scattering (DIS)

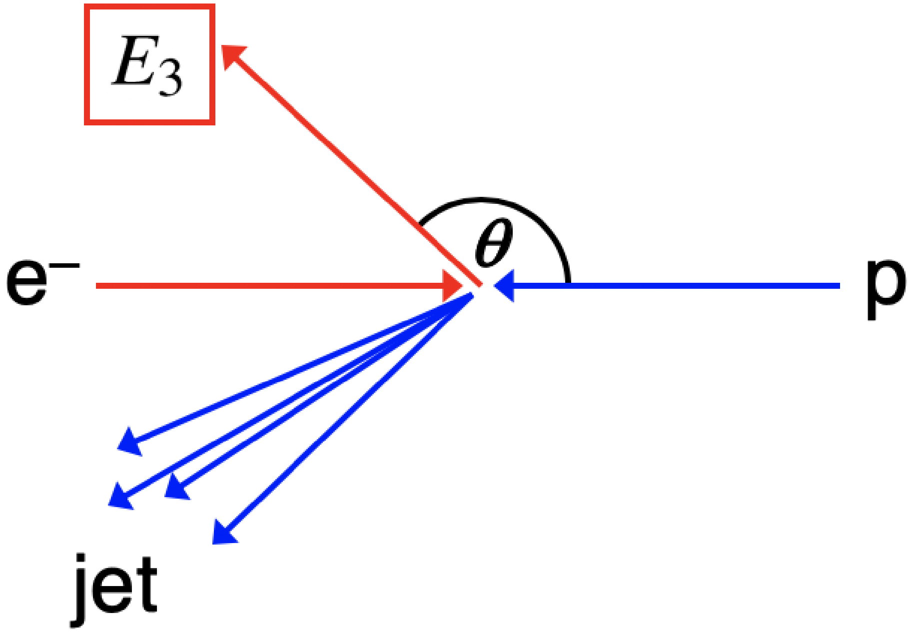
- The Form Factors have been replaced by structure functions: $F_1(Q^2, x)$ and $F_2(Q^2, x)$
  - $F_1(Q^2, x)$ and $F_2(Q^2, x)$ depend on $x \Longrightarrow$ can't be interpreted as Fourier transforms of the proton charge and magnetic moment distributions but describe the momentum distribution of the quarks inside the proton!
  - $F_1(Q^2, x)$: magnetic interaction
  - $F_2(Q^2, x)$: electric and magnetic interactions
- For Deep Inelastic Scattering at very high energies $\left(Q^2 \gg m_p^2 y^2\right)$ we get

$$\frac{d^2\sigma}{dx\mathrm{d}Q^2} = \frac{4\pi\alpha^2}{Q^4} \cdot \left[(1-y)\frac{F_2(Q^2, x)}{x} + y^2 F_1(Q^2, x)\right]$$

Deep Inelastic Scattering (DIS)

- In the lab. frame it's convenient to express the cross section in terms of the angle θ and energy E_3 of the scattered electron, which can be well-measured experimentally





$$Q^2 = 4E_1E_3\sin^2\frac{\theta}{2}, \qquad x = \frac{Q^2}{2m_p(E_1 - E_3)}, \qquad y = 1 - \frac{E_3}{E_1}, \qquad \nu = E_1 - E_3$$

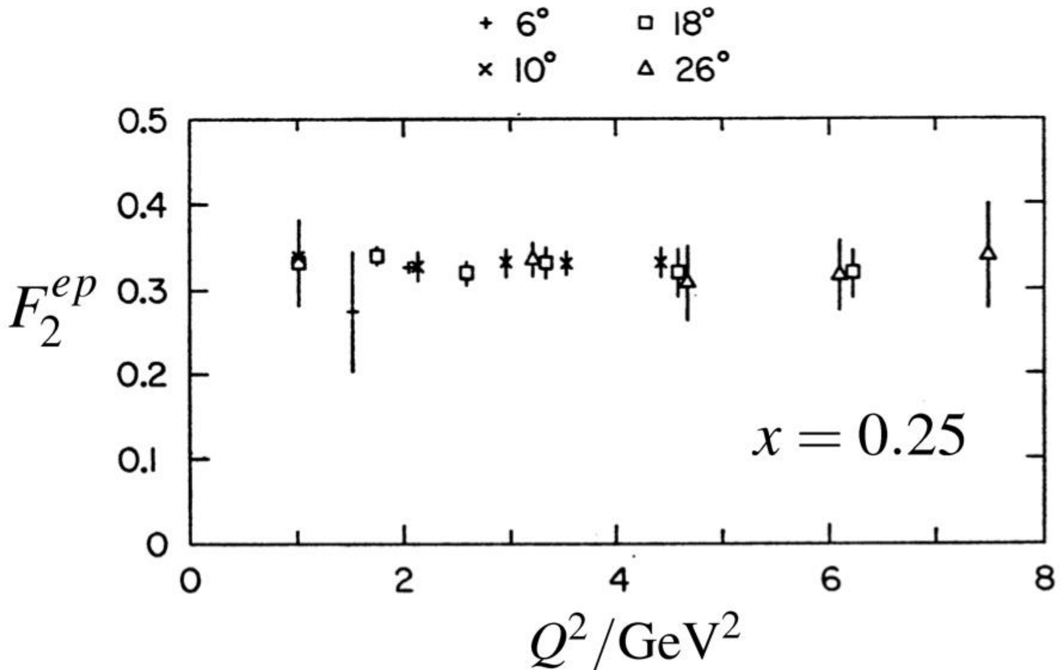
Deep Inelastic Scattering (DIS)

- In the lab. frame we get

pure magnetic structure function

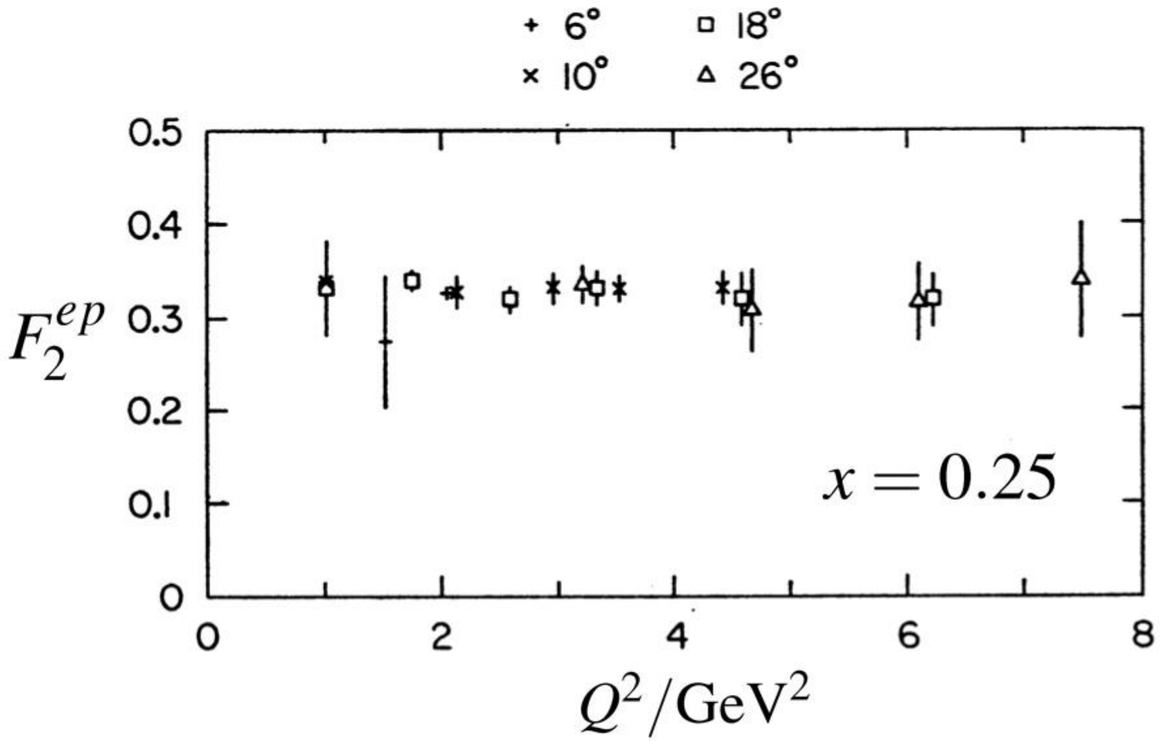
$$\frac{d^2\sigma}{dE_3 d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4\theta/2}\left(\frac{1}{\nu}F_2(Q^2,x)\cos^2\frac{\theta}{2} + \frac{2}{m_p}F_1(Q^2,x)\sin^2\frac{\theta}{2}\right)$$

electric + magnetic structure function



Measuring the structure functions

- To determine $F_2(Q^2, x)$ and $F_1(Q^2, x)$ for a given x and Q^2 we need measurements of the differential cross section at several different scattering angles and incoming electron beam energies
- Example: the distribution of F_2 vs Q^2 for electron-proton scattering at fixed x



Friedman, Kendall, Ann. Rev. Nucl. Part. Sci. 22 (1972) 203

- It is observed experimentally that both F_1 and F_2 are (almost) independent of Q^2

Bjorken scaling and Callan-Gross relation

- The near independence of the structure functions on Q^2 is known as Bjorken scaling

$$F_1(Q^2, x) \to F_1(x), \qquad F_2(Q^2, x) \to F_2(x)$$

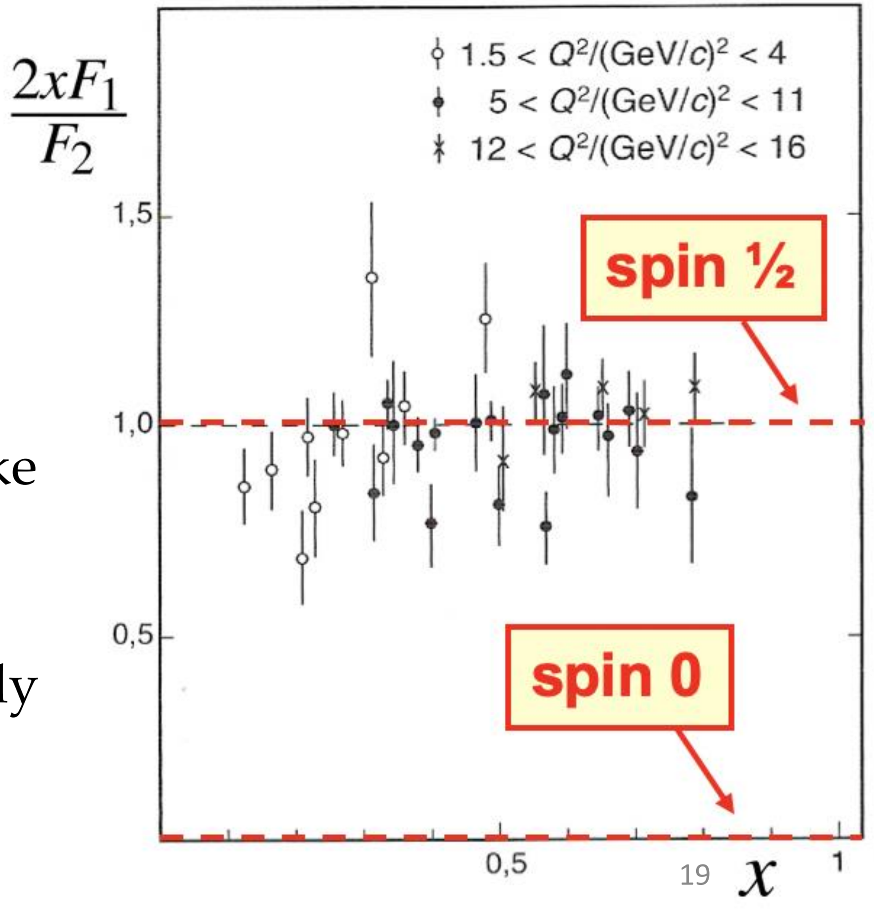
- Highly suggestive of elastic scattering from point-like constituents within the proton

- It is also observed that $F_1(x)$ and $F_2(x)$ are not independent but satisfy the Callan-Gross relation:

$$F_2(x) = 2xF_1(x)$$

- This is exactly what you would expect for scattering from point-like quarks inside the proton

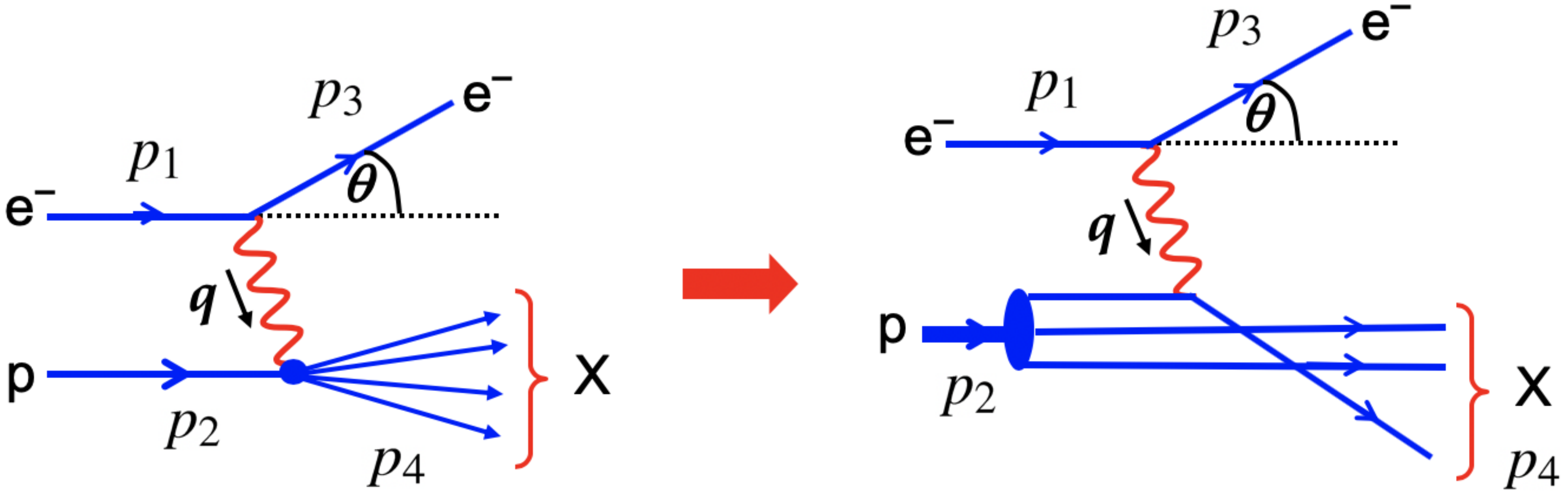
- *Note:* if quarks were spin-zero particles, we would expect the purely magnetic structure function to be zero (i.e. $F_1(x) = 0$)!



The quark-parton model

- Before quarks and gluons were generally accepted, Feynman proposed that the proton was made up of point-like constituents which he called "partons"

- Both Bjorken scaling and Callan-Gross relationship can be explained by assuming that DIS is dominated by the scattering of a single virtual photon from point-like spin-half constituents of the proton

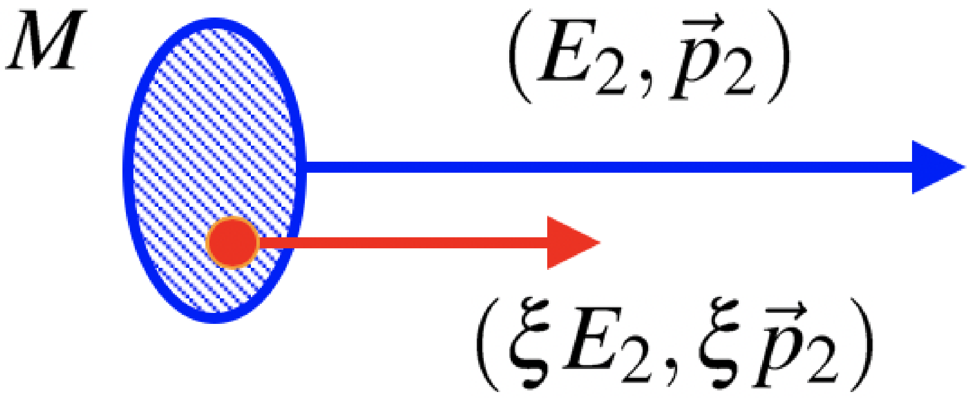


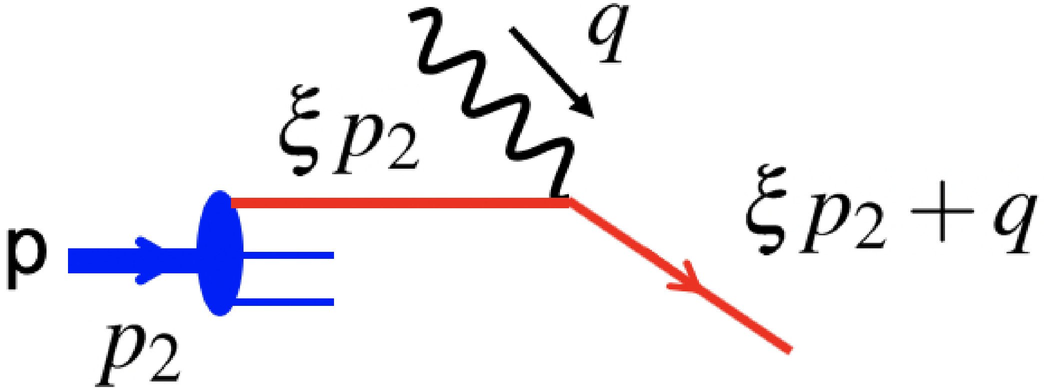
Scattering from a proton with structure functions → **Scattering from a point-like quark within the proton**

- **How do these two pictures of the interaction relate to each other?**

The quark-parton model

- In the parton model the basic interpretation is elastic scattering from "quasi-free" spin-half quarks in the proton (i.e. quark as a free particle)
- The parton model is most easily formulated in a frame where the proton has a very high energy, often referred to as "infinite momentum frame", where we can neglect the proton mass and $p_2 = (E_2, 0{,}0, E_2)$
- In this frame we can also neglect the mass of the quark and any momentum transverse to the direction of the proton
- Let the quark carry a fraction ξ of the proton's four-momentum





The quark-parton model

- After the interaction, the struck quark's four-momentum is $\xi p_2 + q$

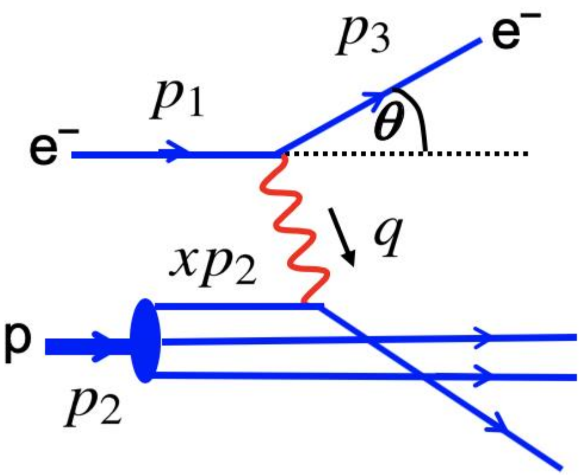
$$(\xi p_2 + q)^2 = m_q^2 \approx 0 \Longrightarrow q^2 + 2\xi p_2 \cdot q = 0 \Longrightarrow \xi = \frac{Q^2}{2p_2 \cdot q} = x$$

- Bjorken x can be identified as the fraction of the proton momentum carried by the struck quark (in a frame where the proton has a very high energy)

- In terms of the proton momentum:

$$s = (p_1 + p_2)^2 \approx 2p_1 \cdot p_2, \qquad y = \frac{p_2 \cdot q}{p_2 \cdot p_1}, \qquad x = \frac{Q^2}{2p_2 \cdot q}$$

The quark-parton model



- After the interaction, the struck quark's four-momentum is $\xi p_2 + q$

$$s_q = (p_1 + xp_2)^2 = 2xp_1p_2 = xs$$

$$y_q = \frac{p_q \cdot q}{p_q \cdot p_1} = \frac{xp_2 \cdot q}{xp_2 \cdot p_1} = y$$

$x_q = 1$ (elastic, quark does not break up)

- Previously derived LI cross section for $e^-\mu^- \to e^-\mu^-$ elastic scattering in the ultra-relativistic limit applied to $e^-q \to e^-q$ scattering:

$$\frac{d\sigma}{dq^2} = \frac{2\pi\alpha^2 e_q^2}{q^4} \cdot \left[1 + \left(1 + \frac{q^2}{s_q}\right)^2\right], e_q = +2/3(-1/3)$$

- Using $Q^2 = -q^2 = (s_q - m_q^2)x_q y_q \Longrightarrow \frac{q^2}{s_q} = -y_q = -y$

$$\frac{d\sigma}{dQ^2} = \frac{2\pi\alpha^2 e_q^2}{Q^4} \cdot [1 + (1 - y)^2]$$

The quark-parton model

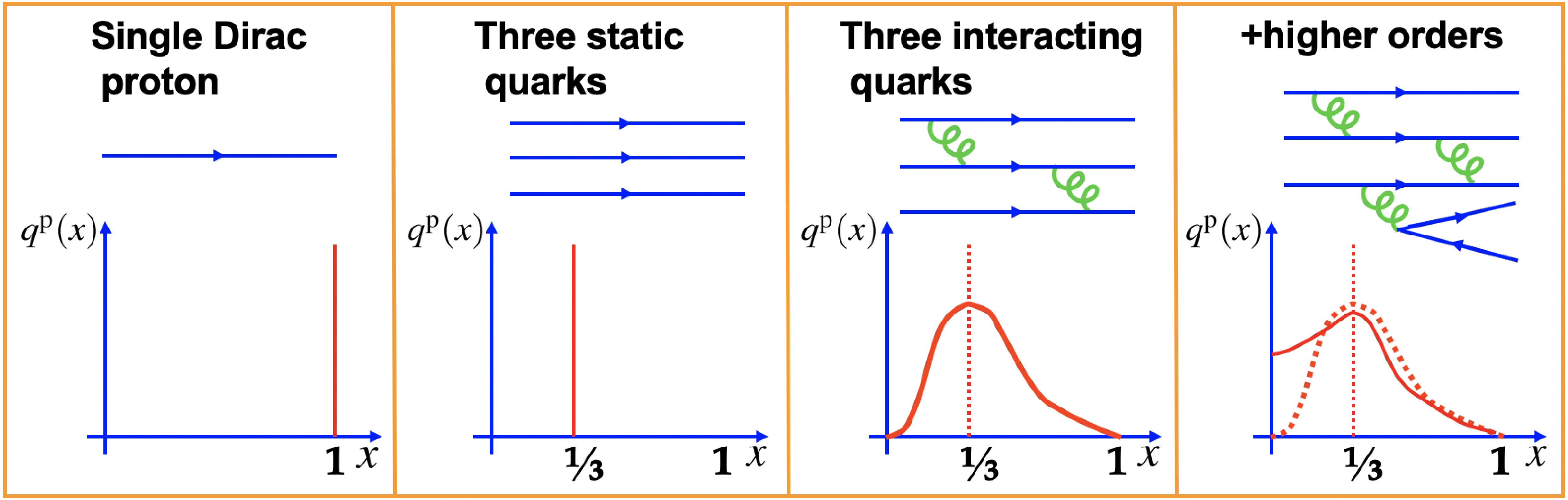
$$\frac{d\sigma}{dQ^2} = \frac{2\pi\alpha^2 e_q^2}{q^4} \cdot \left[1 + (1-y)^2\right]$$

- This is the expression for the differential cross section for elastic $e^- q \rightarrow e^- q$ scattering from a quark carrying a fraction x of the proton momentum
- Now we need to account for the distribution of quark momenta within the proton
- Introduce **parton distribution functions** such that $q^p(x)dx$ is the number of quarks of type q within a proton with momentum fraction between x and $x + dx$

What form would you expect for the parton distribution functions?

The quark-parton model

- Expected form of the parton distribution functions?



The quark-parton model

- The cross section for scattering from a particular quark type within the proton which is in the range from x to $x + dx$:

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4} \cdot \left[(1-y) + \frac{y^2}{2}\right] \times e_q^2 q^p(x) dx$$

- Summing over all types of quarks within the proton gives the expression for the electron-proton scattering cross section:

$$\frac{d^2\sigma^{ep}}{dx dQ^2} = \frac{4\pi\alpha^2}{Q^4} \cdot \left[(1-y) + \frac{y^2}{2}\right] \sum_q e_q^2 q^p(x)$$

The quark-parton model

- Compare with electron-proton scattering cross section in terms of the structure functions

$$\frac{d^2\sigma^{ep}}{dxdQ^2} = \frac{4\pi\alpha^2}{Q^4} \cdot \left[(1-y)\frac{F_2(Q^2,x)}{x} + y^2 F_1(Q^2,x)\right]$$

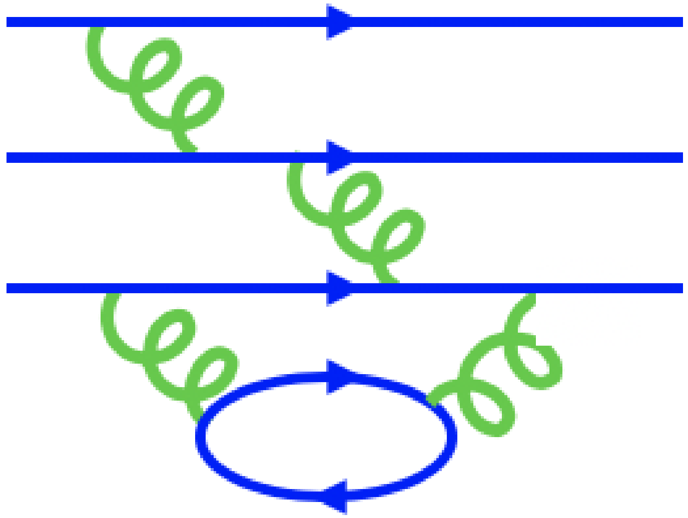
- Comparing the equations from the previous slide we get the parton model prediction for the structure functions in general LI from for the differential cross section

$$F_2^p(Q^2,x) = 2xF_1^p(Q^2,x) = x\sum_q e_q^2 q^p(x)$$

Predictions of the quark-parton model

- **Bjorken scaling:** $F_1(Q^2, x) \to F_1(x),\ F_2(Q^2, x) \to F_2(x)$
  - due to scattering from point-like particles within the proton
- **Callan-Gross relation:** $F_2(x) \to 2xF_1(x)$
  - due to scattering from spin-half Dirac particles inside the proton
  - the magnetic moment is directly related to the charge $\Rightarrow$ the "electro-magnetic" and "pure magnetic" structure functions are fixed with respect to each other
- At present parton distribution functions can't be calculated from QCD
  - can`t use perturbation theory because the strong coupling constant, α_s, is large
- Measurement of the structure functions enable us to determine the parton distribution functions

Predictions of the quark-parton model



- For electron-proton scattering we have

$$F_2^p(x) = x \sum_q e_q^2 q^p(x)$$

- Due to higher order effects, the proton contains not only up and down quarks but also anti-up, anti-down
    - for now, we will neglect the small contributions from heavier quarks

Predictions of the quark-parton model

- For electron-proton scattering we have

$$F_2^{ep}(x) = x\sum_q e_q^2 q^p(x) = x\left(\frac{4}{9}u^p(x) + \frac{1}{9}d^p(x) + \frac{4}{9}\bar{u}^p(x) + \frac{1}{9}\bar{d}^p(x)\right)$$

- For electron-neutron scattering we have

$$F_2^{en}(x) = x\sum_q e_q^2 q^n(x) = x\left(\frac{4}{9}u^n(x) + \frac{1}{9}d^n(x) + \frac{4}{9}\bar{u}^n(x) + \frac{1}{9}\bar{d}^n(x)\right)$$

- We can assume "isospin symmetry", the neutron (ddu) is the same a proton (uud) with up and down quarks interchanged

$$d^n(x) = u^p(x), \qquad u^n(x) = d^p(x)$$

and define the neutron distribution functions in terms of those of the proton

$$u(x) \equiv u^p(x) = d^n(x), \qquad d(x) = d^p(x) = u^n(x)$$

$$\bar{u}(x) \equiv \bar{u}^p(x) = \bar{d}^n(x), \qquad \bar{d}(x) = \bar{d}^p(x) = \bar{u}^n(x)$$

Predictions of the quark-parton model

- Which gives:

$$F_2^{ep}(x) = 2xF_1^{ep}(x) = x\left(\frac{4}{9}u(x) + \frac{1}{9}d(x) + \frac{4}{9}\bar{u}(x) + \frac{1}{9}\bar{d}(x)\right)$$
$$F_2^{en}(x) = 2xF_1^{en}(x) = x\left(\frac{4}{9}d(x) + \frac{1}{9}u(x) + \frac{4}{9}\bar{d}(x) + \frac{1}{9}\bar{u}(x)\right)$$

- Integrating the above equations, we get

$$\int_0^1 F_2^{ep}(x)dx = \int_0^1 x\left(\frac{4}{9}[u(x) + \bar{u}(x)] + \frac{1}{9}\left[d(x) + \bar{d}(x)\right]\right)dx = \frac{4}{9}f_u + \frac{1}{9}f_d$$

$$\int_0^1 F_2^{en}(x)dx = \int_0^1 x\left(\frac{4}{9}\left[d(x) + \bar{d}(x)\right] + \frac{1}{9}[u(x) + \bar{u}(x)]\right)dx = \frac{4}{9}f_d + \frac{1}{9}f_u$$

- $f_u = \int_0^1 [xu(x) + x\bar{u}(x)]\, dx$ is the fraction of the proton momentum carried by the up and anti-up quarks

Predictions of the quark-parton model

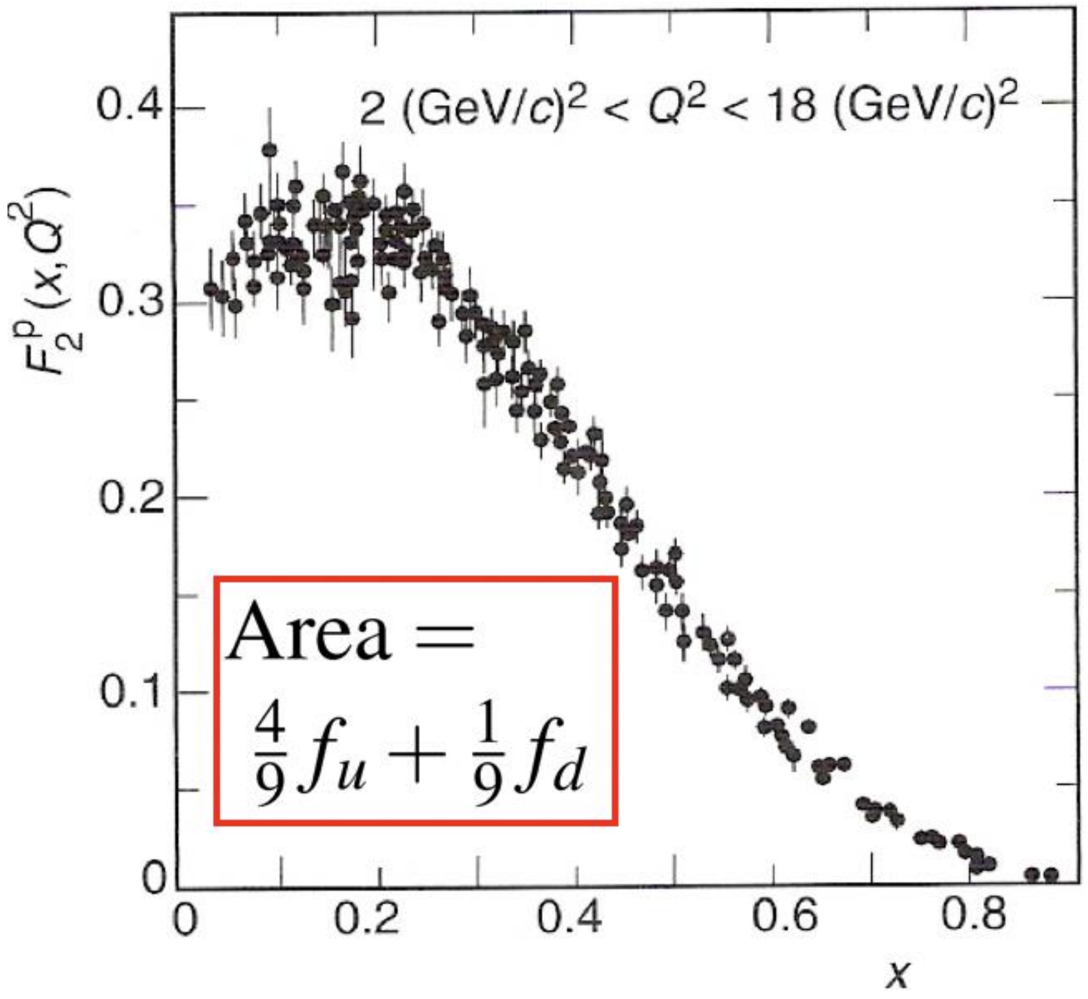
- <u>Experimentally:</u>

$$\int F_2^{ep}(x)dx \approx 0.18$$

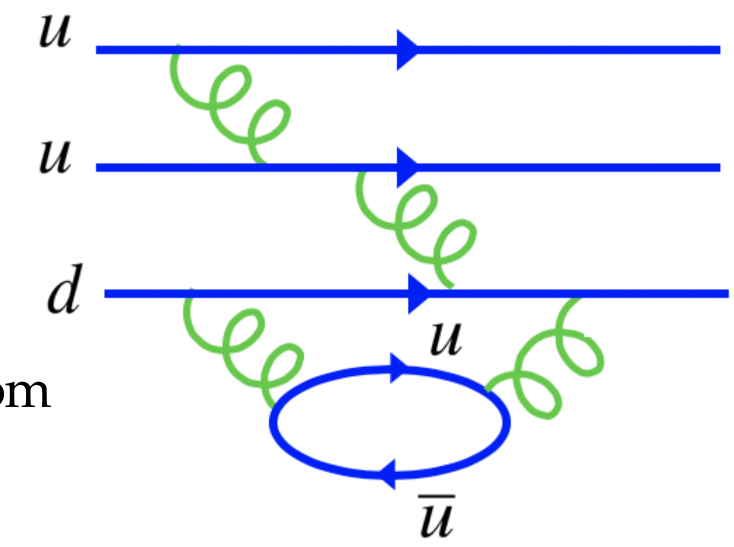
$$\int F_2^{en}(x)dx \approx 0.12$$

$$\Rightarrow f_u \approx 0.36, f_d \approx 0.18$$

- In the proton, as expected, the up quarks carry twice the momentum compared to the down quarks
- The quarks carry just over 50% of the total proton momentum
- The rest is carried by gluons (gluons are neutral and don`t contribute to electron-nucleon scattering!)



Valence and sea quarks



- As we are beginning to see, the proton is complex!
- The parton distribution function $u^p(x) = u(x)$ includes contributions from the "valence" quarks and virtual quarks produced by gluons: **the "sea"**
- Resolving into valence and sea contributions:

$$u(x) = u_V(x) + u_S(x) \qquad d(x) = d_V(x) + d_S(x)$$

$$\bar{u}(x) = \bar{u}_S(x) \qquad \bar{d}(x) = \bar{d}_S(x)$$

- The proton contains two valence up quarks and one valence down quark so we would expect

$$\int_0^1 u_V(x)dx = 2\,, \qquad \int_0^1 d_V(x)dx = 1$$

- **No a priori expectation for the total number of sea quarks!**

Valence and sea quarks

- Sea quarks arise from gluon quark-antiquark pair production and with $m_u = m_d$ it is reasonable to expect

$$u_S(x) = d_S(x) = \bar{u}_S(x) = \bar{d}_S(x) = S(x)$$

- Using this relation, we can obtain for $F_2^{ep}(x)$ and $F_2^{en}(x)$ from slide 31

$$F_2^{ep}(x) = x\left(\frac{4}{9}u_V(x) + \frac{1}{9}d_V(x) + \frac{10}{9}S(x)\right)$$

$$F_2^{en}(x) = x\left(\frac{4}{9}d_V(x) + \frac{1}{9}u_V(x) + \frac{10}{9}S(x)\right)$$

- Which gives the ratio

$$\frac{F_2^{en}(x)}{F_2^{ep}(x)} = \frac{4d_V(x) + u_V(x) + 10S(x)}{4u_V(x) + d_V(x) + 10S(x)}$$

Valence and sea quarks

- The sea component arise from processes such as $g \to u\bar{u}(d\bar{d}, s\bar{s}, \dots)$
- Due to the $1/q^2$ dependence of the gluon propagator, much more likely to produce low-energy gluons
- We expect the sea to comprise low-energy $q/\bar{q}$
- Therefore, at low x we expect the sea quarks to dominate:

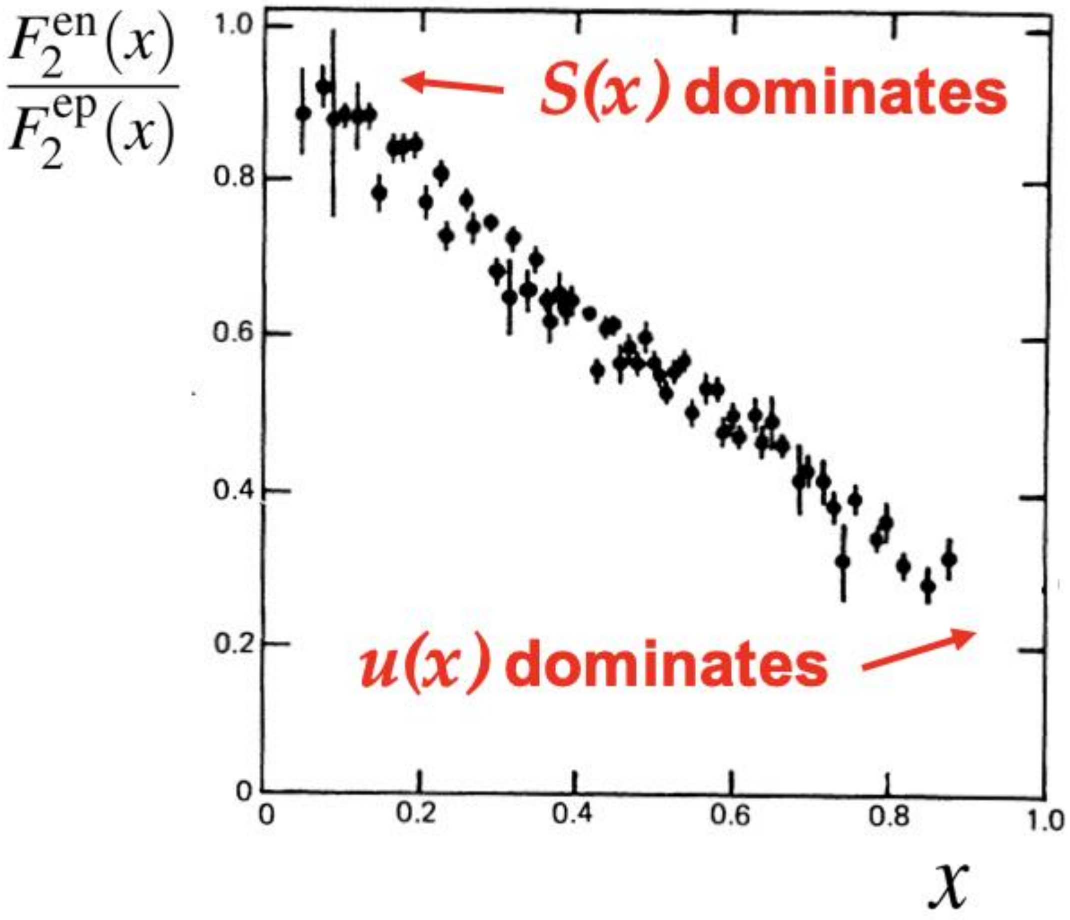
$$\frac{F_2^{en}(x)}{F_2^{ep}(x)} \to 1 \text{ as } x \to 0$$

Observed experimentally

- At high x we expect the sea contribution to be small

$$\frac{F_2^{en}(x)}{F_2^{ep}(x)} = \frac{4d_V(x) + u_V(x)}{4u_V(x) + d_V(x)} \text{ as } x \to 1$$

Note: $u_V = 2d_V$ would give ratio $2/3$ as $x \to 1$



Valence and sea quarks

Experimentally we observe:

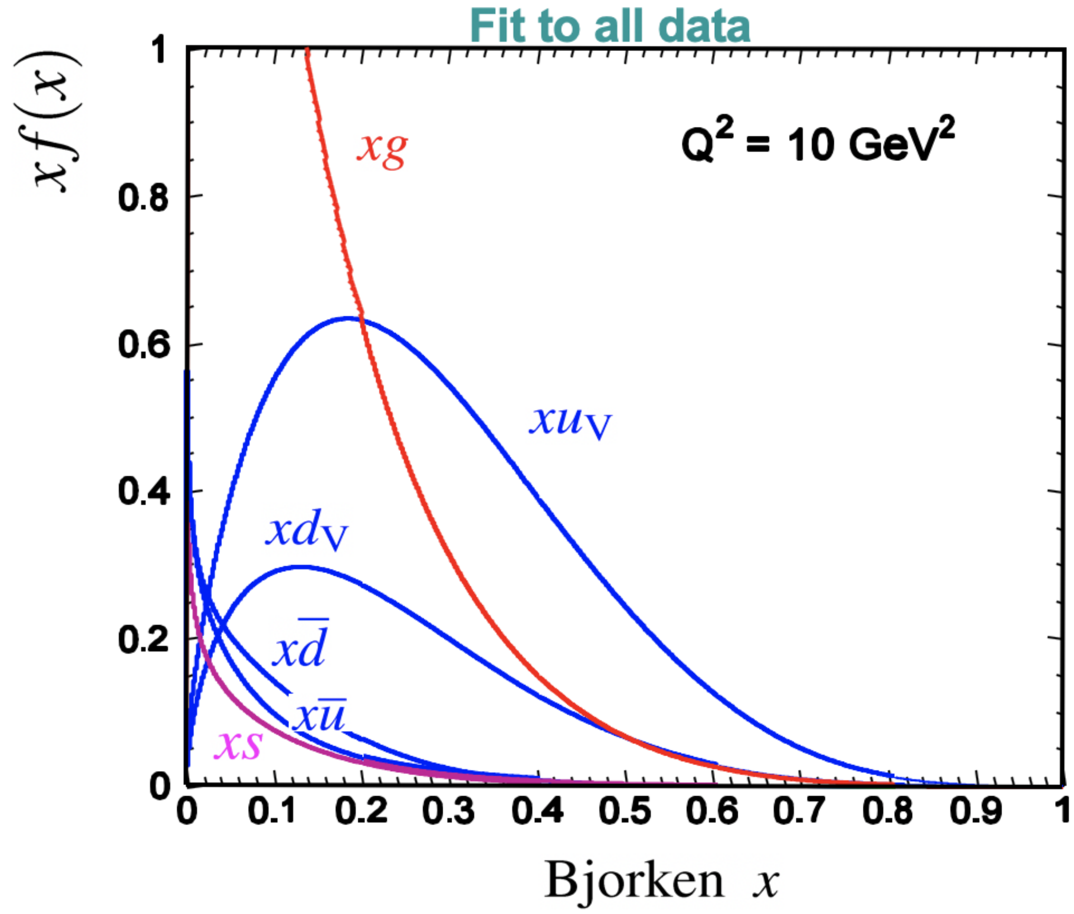
$$F_2^{en}(x)/F_2^{ep}(x) \rightarrow 1/4 \text{ as } x \rightarrow 1$$

$$\Longrightarrow d(x)/u(x) \rightarrow 0 \text{ as } x \rightarrow 1$$

This behaviour is not well understood

Parton distribution functions (PDF)

- Ultimately the parton distribution functions are obtained from a fit to all experimental data including neutrino scattering
- Hadron-hadron collisions give information about the gluon PDF $g(x)$

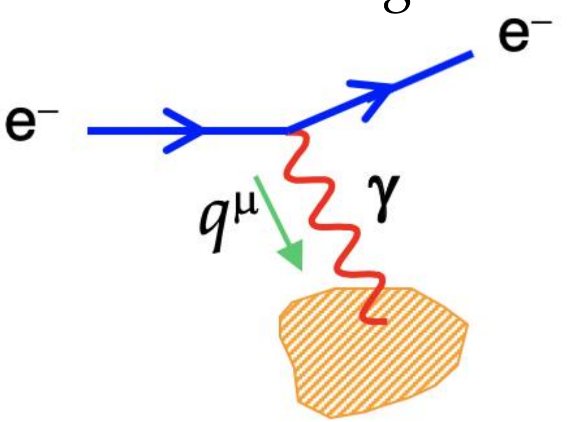


- Apart from large x: $u_V(x) \approx d_V(x)$
- For $x < 0.2$ gluons dominate
- In fits to data assume $u_S(x) = \bar{u}(x)$
- $\bar{d}(x) > \bar{u}(x)$ not understood – exclusion principle?
- Small strange quark component $s(x)$

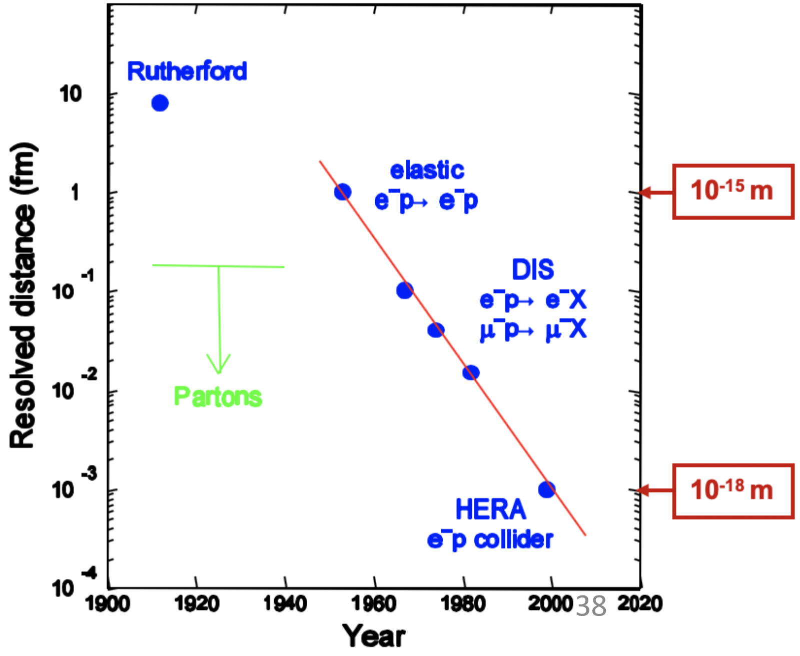
Scaling violations

- In the last 40 years, experiments have probed the proton with virtual photons of ever-increasing energy
- The non-point like nature of the scattering becomes apparent when $\lambda_\gamma \sim$ size of the scattering centre

$$\lambda_\gamma = \frac{h}{|\vec{q}|} \sim \frac{1\ \mathrm{GeV \cdot fm}}{|\vec{q}|(\mathrm{GeV})}$$



- Scattering from point-like quarks gives rise to Bjorken scaling (no q^2 cross section dependence)
- If quarks were not point-like, at high q^2 (when $\lambda_\gamma \sim$ size of a quark) we would observe rapid decrease in cross section with increasing q^2
- To search for quark sub-structure we need to go to highest q^2: **HERA**

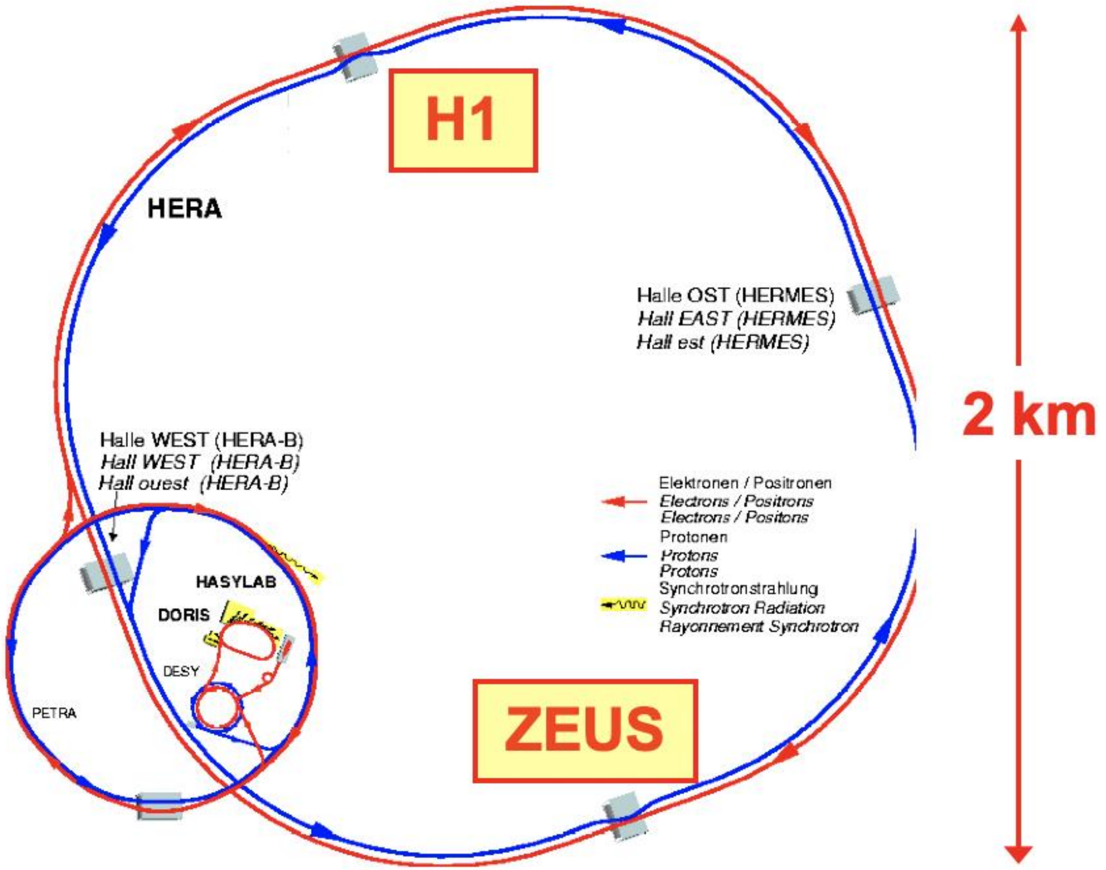


HERA $e^{\pm}p$ collider: $\mathbf{1991-2007}$

★ **DESY (Deutsches Elektronen-Synchroton) Laboratory, Hamburg, Germany**

e± **27.5 GeV** → ← **820 GeV** **p**

$\sqrt{s} = 300$ **GeV**

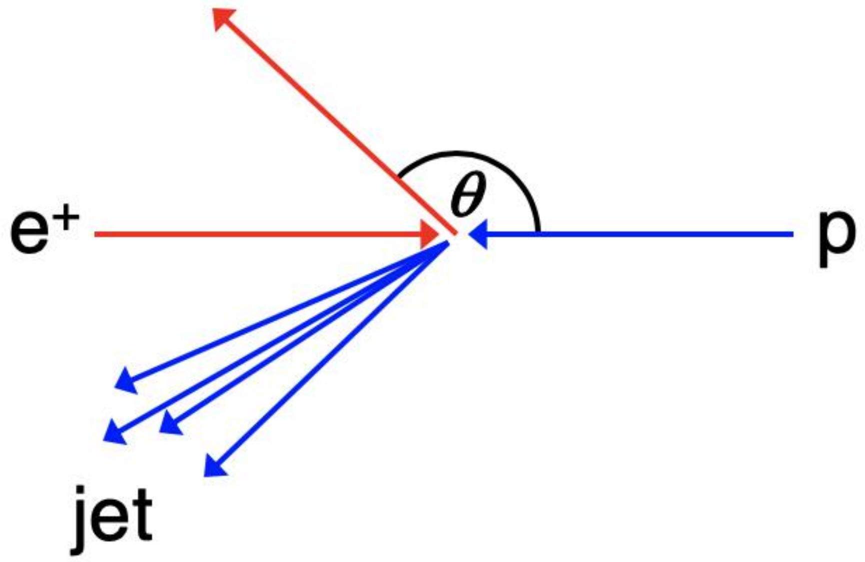


★ **Two large experiments : H1 and ZEUS**

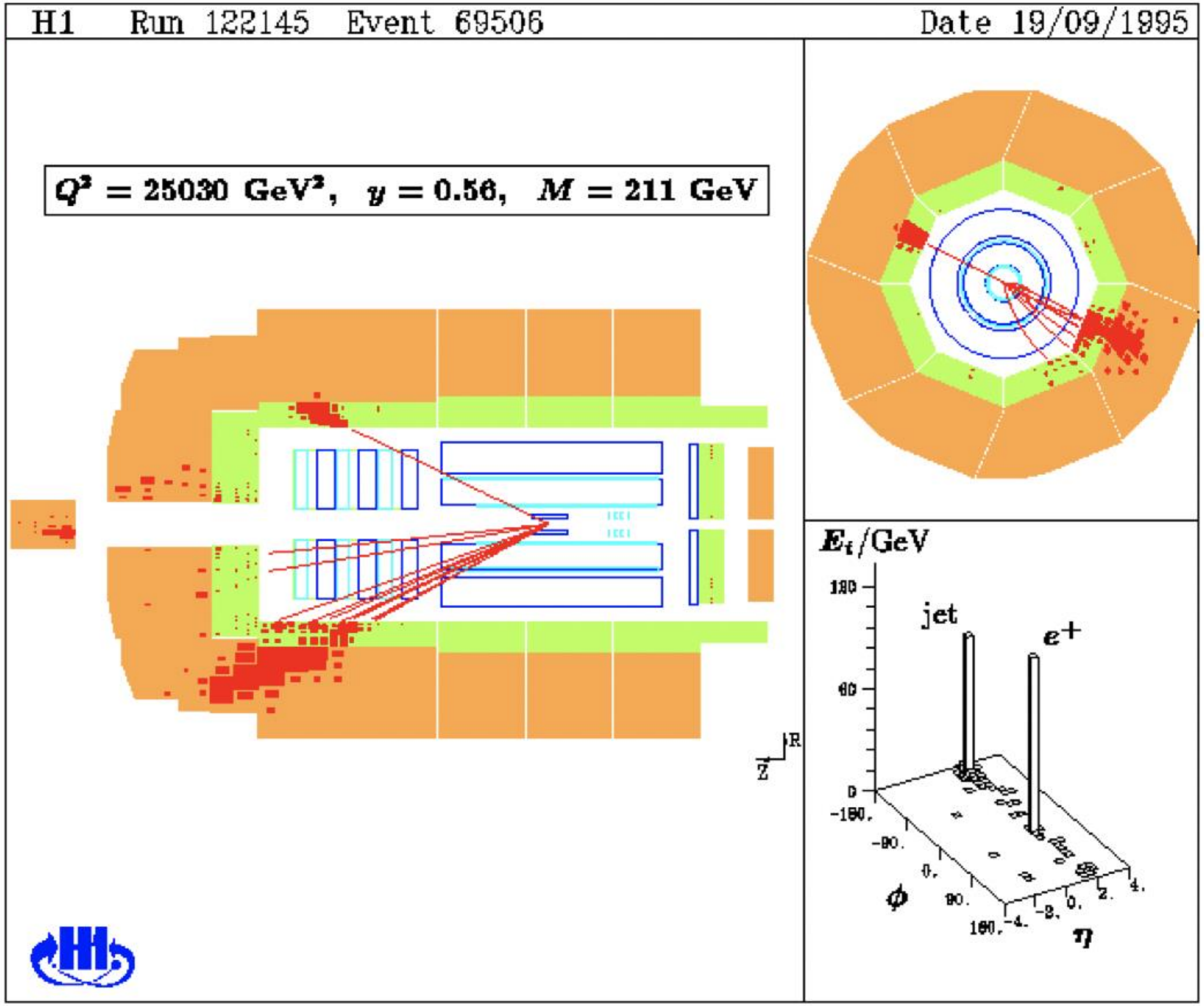
★ **Probe proton at very high Q^2 and very low x**

Examples of high Q^2 event in H1

✶ **Event kinematics determined from electron angle and energy**



✶ **Also measure hadronic system (although not as precisely) – gives some redundancy**



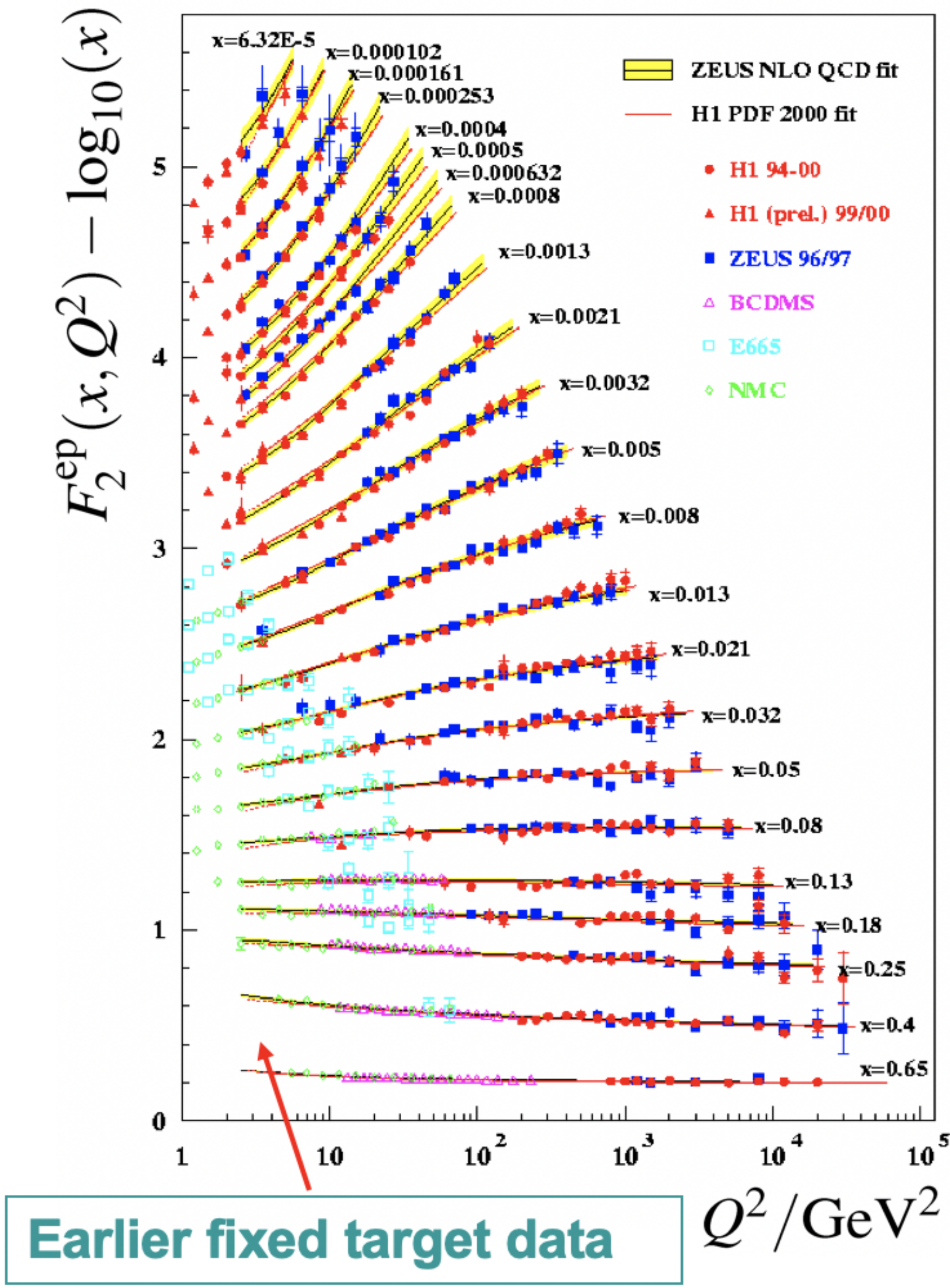
$F_2(Q^2, x)$ results

- No evidence of rapid decrease of cross section at highest Q^2

$$\Longrightarrow R_{\text{quark}} < 10^{-18}\ \text{m}$$

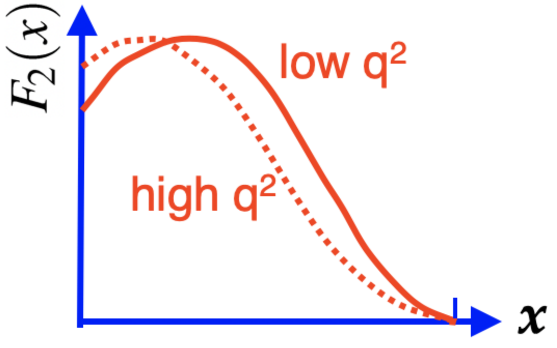
- For $x > 0.05$, only weak dependence of F_2 on Q^2
  - consistent with the expectations from the quark-parton model
- We observe clear scaling violations, particularly at low x:

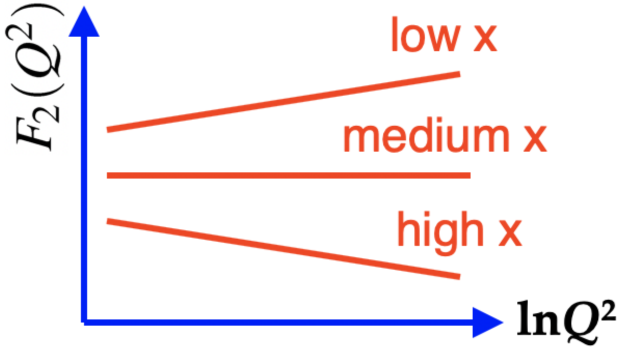
$$F_2(Q^2, x) \neq F_2(x)$$



Origin of scaling violations

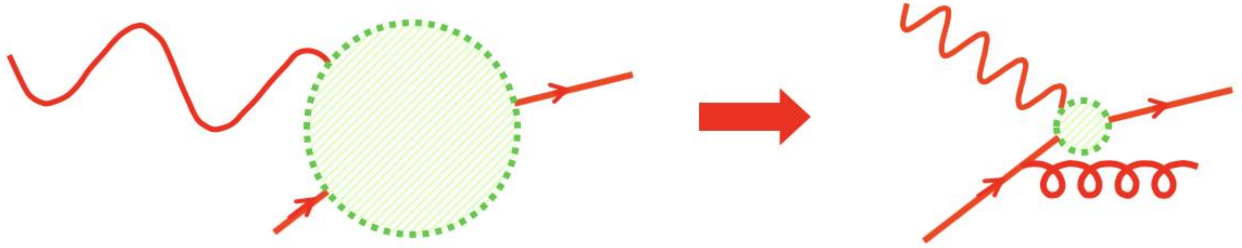
- Observe "small" deviations from exact Bjorken scaling $F_2(x) \rightarrow F_1(x, Q^2)$





- At high Q^2 we observe more low-x quarks

- Explanation: at high Q^2 (shorter wavelength) resolve finer structure: reveal that quarks are sharing momentum with gluons (at high Q^2 we expect to "see" more low-x quarks)

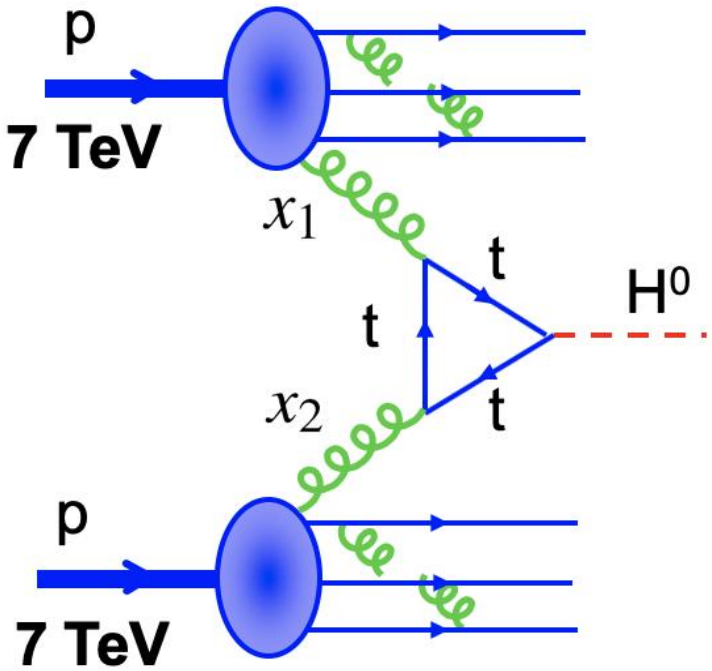


- Even though QCD can't predict the exact x dependence of $F_2(Q^2, x)$, the observed scaling violations in DIS are a powerful validation of the fundamental QCD theory of strong interactions!

Proton-proton collisions at the LHC

- Measurement of the structure functions not only provides a powerful test of QCD but also the parton distribution functions are essential for the cross section calculations at pp and $p\bar{p}$ colliders

- <u>Example:</u> Higgs production at the Large Hadron Collider (LHC)
  - LHC collided 6.5 TeV protons on 6.5 TeV protons
  - Underlying collisions are between partons
  - Higgs boson production at the LHC dominated by "gluon-gluon" fusion



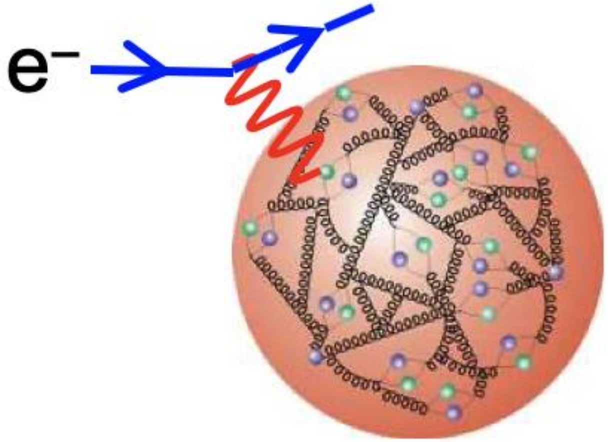
- The cross section depends on the gluon PDF

$$\sigma(pp \to H) \sim \int_0^1 \int_0^1 g(x_1) g(x_2) \sigma(gg \to H) dx_1 dx_2$$

- Uncertainty in the gluon PDFs leads to a $\pm 5\%$ uncertainty in Higgs boson production cross section (now approaching per cent level)

- Prior to HERA data uncertainty was $\pm 25\%$

Summary of Lecture 10



Main learning outcomes

- At very high electron energies $\lambda \ll r_p$ the proton appears to be a sea of quarks and gluons
- Deep Inelastic Scattering (DIS) = Elastic scattering from the quasi-free constituent quarks
  - Bjorken scaling: $F_1(Q^2, x) \rightarrow F_1(x)$ point-like scattering
  - Callan-Gross relation: $F_2(Q^2, x) = 2xF_1(x)$ scattering from spin-half particles
- Describe scattering in terms of parton distribution function $u(x), d(x), \ldots$ which describe the momentum distribution inside a nucleon
- The proton is much more complex than just uud – sea of quarks, antiquarks, and gluons
- Quarks carry only about 50% of the momentum of the proton – the rest is due to low-energy gluons